# Fluid Power System Dynamics

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CENTER FOR COMPACT AND EFFICIENT
FLUID POWER

A National Science Foundation
Engineering Research Center

FLUID POWER SYSTEM DYNAMICS

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Version: September 25, 2015

# Contents

# Preface

This book was created because system dynamics courses in the standard mechanical engineering curriculum do not cover fluid power, even though fluid power is essential to mechanical engineering and students entering the work force are likely to encounter fluid power systems in their job. Most system dynamics textbooks have a chapter or part of a chapter on fluid power but typically the chapter is thin and does not cover practical fluid power as is used in industry today. For example, many textbooks confine their discussion of fluid power to liquid tank systems and never even mention hydraulic cylinders, the workhorse of today's practical fluid power.

The material is intended for use in an introductory system dynamics course that would teach analysis of mechanical translational, mechanical rotary and electrical system using differential equations, transfer functions and time and frequency response. The material should be introduced toward the end of the course after the other domains and most of the analysis methods have been covered. It replaces or supplements any coverage of fluid power in the course textbook. The instruction can pick and choose which sections will be covered in class or read by the student. At the University of Minnesota, the material is used in course ME 3281, System Dynamics and Control and in ME 4232, Fluid Power Control Lab.

The book is a result of the Center for Compact and Efficient Fluid Power (CCEFP) (www.ccefp.org), a National Science Foundation Engineering Research Center founded in 2006. CCEFP conducts basic and applied research in fluid power with three thrust areas: efficiency, compactness, and usability. CCEFP has over 50 industrial affiliates and its research is ultimately intended to be used in next generation fluid power products. To ensure the material in the book is current and relevant, it was reviewed by industry representatives and academics affiliated with CCEFP.

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# 1. Introduction

## 1.1. Overview

Fluid power is the transmission of forces and motions using a confined, pressurized fluid. In hydraulic fluid power systems the fluid is oil, or less commonly water, while in pneumatic fluid power systems the fluid is air.

Fluid power is ideal for high speed, high force, high power applications. Compared to all other actuation technologies, including electric motors, fluid power is unsurpassed for force and power density and is capable of generating extremely high forces with relatively lightweight cylinder actuators. Fluid power systems have a higher bandwidth than electric motors and can be used in applications that require fast starts, stops and reversals, or that require high frequency oscillations. Because oil has a high bulk modulus, hydraulic systems can be finely controlled for precision motion applications. [1] Another major advantage of fluid power is compactness and flexibility. Fluid power cylinders are relatively small and light for their weight and flexible hoses allows power to be snaked around corners, over joints and through tubes leading to compact packaging without sacrificing high force and high power. A good example of this compact packaging are Jaws of Life rescue tools for ripping open automobile bodies to extract those trapped within.

Fluid power is not all good news. Hydraulic systems can leak oil at connections and seals. Hydraulic power is not as easy to generate as electric power and requires a heavy, noisy pump. Hydraulic fluids can cavitate and retain air resulting in spongy performance and loss of precision. Hydraulic and pneumatic systems become contaminated with particles and require careful filtering. The physics of fluid power is more complex than that of electric motors which makes modeling and control more challenging. University and industry researchers are working hard not only to overcome these challenges but also to open fluid power to new applications, for example tiny robots and wearable power-assist tools.

[1] While conventional thinking was that pneumatics were not useful for precision control, recent advances in pneumatic components and pneumatic control theory has opened up new opportunities for pneumatics in precision control.


**Figure 1.1.:** Caterpillar 797B mining truck. Source: Caterpillar

## 1.2. Fluid Power Examples

Fluid power is pervasive, from the gas spring that holds you up in the office chair you are sitting on, to the air drill used by dentists, to the brakes in your car, to practically every large agriculture, construction and mining machine including harvesters, drills and excavators.

The Caterpillar 797B mining truck is the largest truck in the world at 3550 hp (Fig. 1.1). It carries 400 tons at 40 mph, uses 900 g of diesel per 12 hr shift, costs about $6M and has tires that are about $60,000 each. It is used in large mining operations such as the Hull-Rust-Mahoning Open Pit Iron Mine, the world's largest open pit iron mine, located in Hibbing MN and the Muskeg River Mine in Alberta Canada.[2] The 797B uses fluid power for many of its internal actuation systems, including lifting the fully loaded bed.

Shultz Steel, an aerospace company in South Gate CA, has a 40,000-ton forging press that weighs over 5.2 million pounds (Fig. 1.2). It is the largest press in the world and is powered by hydraulics operating at 6,600 psi requiring 24 700 hp pumps.

The Multi-Axial Subassemblage Testing (MAST) Laboratory is located at the University of Minnesota and is used to conduct three-dimensional, quasi-static testing of large scale civil engineering structures, including

[2]"New Tech to Tap North America's Vast Oil Reserves", Popular Mechanics, March 2007.

**Figure 1.2.:** 40,000 ton forging press. Source: Shultz Steel.

buildings, to determine behavior during earthquakes (Fig. 1.3). The MAST system, constructed by MTS Systems, has eight hydraulic actuators that can each push or pull with a force of 3910 kN.

The Caterpillar 345C L excavator is used in the construction industry for large digging and lifting operations and has a 345 hp engine (Fig. 1.4). The 345C L operates at a hydraulic pressure of 5,511 psi to generate a bucket digging force of 60,200 lbs and a lift force of up to 47,350 lbs.

A feller buncher is a large forestry machine that cuts trees in place (Fig. 1.5).

Some of the fastest roller coasters in the world get their initial launch from hydraulics and pneumatics. (Fig. 1.6). Hydraulic launch assist systems pump hydraulic fluid into a bank of accumulators storing energy as a compressed gas. At launch, the energy is suddenly released into a hydraulic motor whose output shaft drives a cable drum with the cable rapidly bringing the train from rest to very high velocities. The Kingda Ka at Six Flags Great Adventure uses this launch and reaches 128 mph in 3.5 s. The Hypersonic SLC at Kings Dominion ups the ante with a compressed air launch system that accelerates riders to 81 mph in 1.8 s.

Most automatic transmissions have hydraulically actuated clutches and bands to control the gear ratios. Fluid is routed through internal passageways in the transmission case rather than through hoses (Fig. 1.7)

The dental drill is used to remove small volumes of decayed tooth

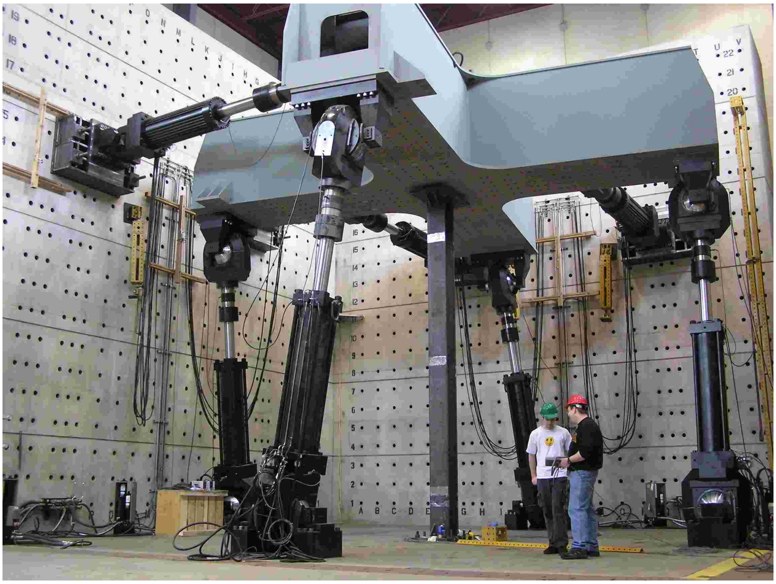

**Figure 1.3.:** MAST Laboratory for earthquake simulation. Source: MAST Lab.

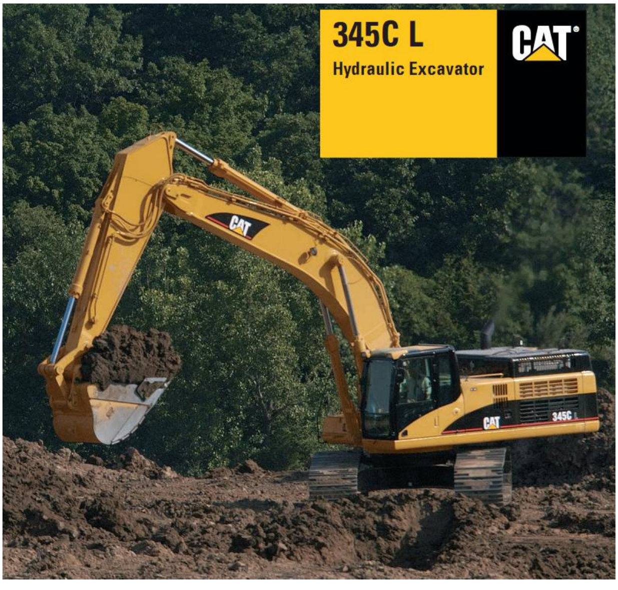


**Figure 1.4.:** Caterpillar 345C L excavator. Source: Caterpillar.


**Figure 1.5.:** Feller buncher. Souce: Wikipedia image.

**Figure 1.6.:** Hypersonic XLC roller coaster with hydraulic lanuch assist. Source: Wikipedia image.

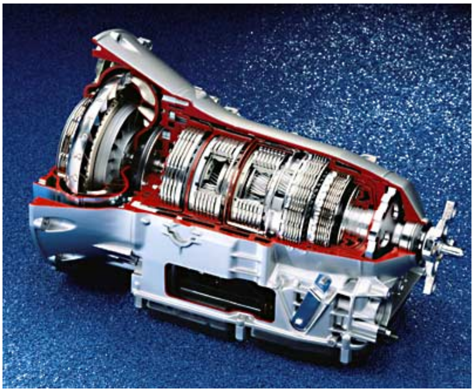

**Figure 1.7.:** Mercedes-Benz automatic transmission model.

prior to inserting a filling (Fig. 1.8). Modern drills rotate at up to 500,000 rpm using an air turbine and use a burr bit for cutting. The hand piece can cost up to $800. Pneumatic drills are used because they are smaller, lighter and faster than electric motor drills. The compressor is located away from the drill and pressurized air is piped to the actuator.

Hydraulic microdrives are used during surgery to position recording electrodes in the brain with micron accuracy (Fig. 1.9). Master and slave cylinders have a 1:1 ratio and are separated by three to four feet of fluid filled cable.

## 1.3. Analyzing Fluid Power Systems

Analyzing the system dynamics of fluid power means using differential equations and simulations to examine the pressures and flows in components of a fluid power circuit, and the forces and motions of the mechanisms driven by the fluid power. For example, in an excavator, the engineer would be interested in determining the diameter and stroke length of the cylinder that is required to drive the excavator bucket and how the force and velocity of the bucket changes with time as the valves

**Figure 1.8.:** A dental handpiece. Source: Wikipedia image.

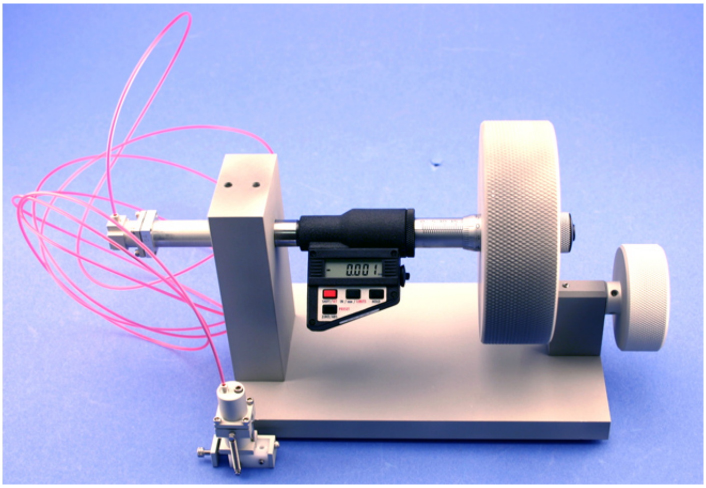

**Figure 1.9.:** Hydraulic microdrive for neural recording electrode placement. Source: Stoelting Co.

to the cylinder are actuated. Because fluid power systems change with time and because fluid power systems have energy storage elements, a dynamic system analysis approach must be taken which means the use of linear and nonlinear differential equations, linear and nonlinear simulations, time responses, transfer functions and frequency analysis.

Fluid power is one domain within the field of system dynamics, just as mechanical translational, mechanical rotational and electronic networks are system dynamic domains. Fluid power systems can be analyzed with the same mathematical tools used to describe spring-mass-damper or inductor-capacitor-resistor systems. Like the other domains, fluid power has fundamental power variables and system elements connected in networks. Unlike other domains many fluid power elements are nonlinear which makes closed-form analysis somewhat more challenging, but not difficult to simulate. Many concepts from transfer functions and basic closed loop control systems are used to analyze fluid power circuits, for example the response of a servovalve used for precision control of hydraulic pistons. [3]

Like all system dynamics domains, fluid power is characterized by two power variables that when multiplied form power, and ideal lumped elements including two energy storing elements, one energy dissipating element, a flow source element and a pressure source element. Table 1.1 shows the analogies between fluid power elements and elements in other domains. Lumping fluid power systems into elements is useful

[3]See "'Transfer Functions for Mood Servovalves"' available on-line in the technical documents section of the Moog company web site.

**Table 1.1.:** Element analogies in several domains.

| *Domain* | *Power Variable 1* | *Power Variable 2* | *Storage Element 1* | *Storage Element 2* | *Dissipative Element* |
|---|---|---|---|---|---|
| Translational | Force, $F$ | Velocity, $V$ | Mass, $M$ | Spring, $K$ | Damper, $B$ |
| Rotational | Torque, $T$ | Velocity, $\omega$ | Inertia, $J$ | Spring, $K$ | Damper, $B$ |
| Electrical | Current, $I$ | Voltage, $V$ | Inductor, $L$ | Capacitor, $C$ | Resistor, $R$ |
| Fluid Power | Flow, $Q$ | Pressure, $P$ | Inertance, $I_f$ | Capacitor, $C_f$ | Resistance, $R_f$ |

when analyzing complex circuits.

# 2. Basic Principles of Fluid Power

## 2.1. Pressure and Flow

Fluid power is characterized by two main variables, *pressure* and *flow*, whose product is power. Pressure $P$ is force per unit area and flow $Q$ is volume per time. Because pneumatics uses compressible gas as the fluid, mass flow rate $Q_m$ is used for the flow variable when analyzing pneumatic systems. For hydraulics, the fluid is generally treated as incompressible, which means ordinary volume flow $Q$ can be used.

Pressure is reported several common units that include pounds per square inch (common engineering unit in the U.S.), pascal (one newton per square meter, the SI unit), megapascal and bar. Table 2.1 shows how pressure units are related. For engineering, it is best to do calculations and simulations in SI units, but to report in SI and the conventional engineering unit.

Pressure is an *across type* variable, which means that it is always measured with respect to a reference just like voltage in an electrical system. As shown in Figure 2.1, one can talk about the pressure across a fluid power element such as a pump or a valve, which is the pressure differential from one side to the other, but when describing the pressure at a point, for example the pressure of fluid at one point in a hose, it is always with respect to a reference pressure. Reporting *absolute pressure* means that the pressure is measured with respect to a perfect vacuum. It is more common to measure and report *gauge pressure*, the pressure relative to ambient atmospheric pressure (0.10132 mPA, 14.7 psi at sea level). The distinction is critical when analyzing the dynamics of pneumatic systems because the ideal gas law that models the behavior of air is based on absolute pressure.

**Table 2.1.:** Conversions between pressure units

| | pascal (Pa) | megapascal (Mpa) | bar (bar) | lbs-sq-in (psi) |
|---|---|---|---|---|
| 1 Pa | 1 | $10^{-6}$ | $10^{-5}$ | $145.04 \times 10^{-6}$ |
| 1 Mpa | $10^6$ | 1 | 10 | 145 |
| 1 bar | $10^5$ | 0.1 | 1 | 14.5 |
| 1 psi | 6895 | $6.895 \times 10^{-3}$ | 0.06895 | 1 |

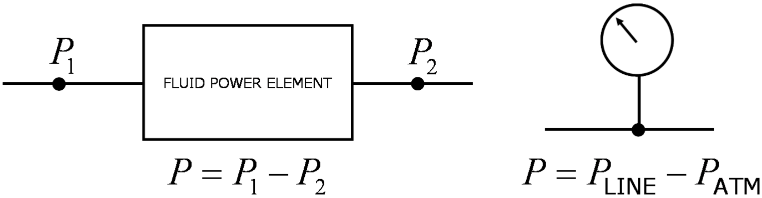


**Figure 2.1.:** Pressure is measured with respect to a reference.

Pressure is measured with a mechanical dial type pressure gage or with an electronic pressure transducer that outputs a voltage proportional to pressure (Fig. 2.2). Almost all pressure transducers report gauge pressure because they expose their reference surface to atmosphere.

Volume flow rate is reported in gallons per minute, liters per minute and cubic meters per second (SI unit). Table 2.2 shows how flow rate units are related.

Flow is a *through type* variable, which means it is volume of fluid flowing through an imaginary plane at one location. Like current in an electrical system, there is no reference point. Flow is measured with a flow meter placed in-line with the fluid circuit. One common type of flow meter contains a turbine, vane or paddle wheel that spins with the flow. Another type has a narrowed passage or an orifice and flow is estimated by measuring the differential pressure across the obstruction. A Pitot tube estimates velocity by measuring the dynamic pressure, which is the difference between the stagnation pressure and static pressure. Figure 2.3 shows some common types of flow meters. Because flow meters restrict the flow, they are used sparingly in systems where small pressure drops matter.

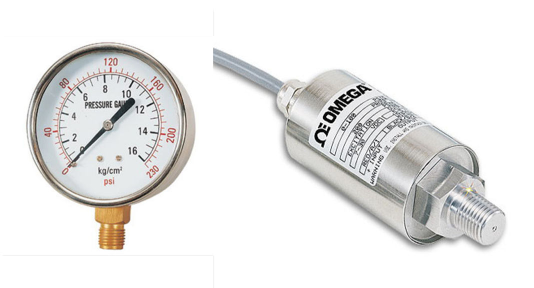


**Figure 2.2.:** Dial pressure gauge and electronic pressure transducer.

**Table 2.2.:** Conversions between volume flow rate units

| | gallon/minute (gpm) | liter/minute (lpm) | cubicmeter/second ($m^3/s$) |
|---|---|---|---|
| 1 gpm | 1 | 3.785 | $6.31 \times 10^{-5}$ |
| 1 lpm | 0.264 | 1 | $1.67 \times 10^{-5}$ |
| 1 $m^3/s$ | $1.585 \times 10^4$ | $6 \times 10^4$ | 1 |

## 2.2. Power and Efficiency

The power available at any one point in a fluid power system is the pressure times the flow at that point

$$power = P \times Q \tag{2.1}$$

For the power available in a conduit, the pressure in Equation 2.1 is the pressure relative to the pressure in the system reservoir, which is typically at atmospheric pressure.

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**Example 2.2.1.** The hose supplying the cylinder operating the bucket of a large excavator has fluid at 1000 psi flowing at 5 gpm. What is the available power in the line?


**Figure 2.3.:** Types of flowmeters. Top row: turbine, digital paddle, variable area. Bottom row is a dual-rotor turbine flowmeter with a cutaway.

*Solution*: For most engineering examples, the reported pressure is a gage pressure, which means the hose is operating at 1000 psi above atmospheric pressure, the pressure of the reservoir. To calculate the power

$$1000 \text{ psi} \times 6895 \text{ Pa/psi} = 6.9{\times}10^6 \text{ Pa}$$
$$5 \text{ gpm} \times 6.31{\times}10^{-5} \text{ m}^3\text{/s/gpm} = 31.6{\times}10^{-5} \text{ m}^3\text{/s}$$
$$\text{Power} = 6.9 \times 10^6 \times 31.6 \times 10^{-5} = 2180 \text{ watts} = 2.9 \text{ horsepower}$$

---

Components such as cylinders, motors and pumps have input and output powers, which can be used to calculate the efficiency of the component. For example, pressured fluid flows into a cylinder and the cylinder extends. The input power is the pressure of the fluid times its flow rate while the output power is the compression force in the cylinder rod times the rod extension velocity. Dividing output power by input power yields the efficiency of the component. The same can be done for components such as an orifice. The efficiency of the orifice is the output pressure divided by the input pressure because the flow rate is the same on either side of the orifice.

## 2.3. Hydraulic Fluids

The main purpose of the fluid in a fluid power system is to transmit power. There are other, practical considerations that dictate the specific fluids used in real hydraulic systems. The fluids must cool the system by dissipation of heat in a radiator or reservoir, must help with sealing to prevent leaks, must lubricate sliding and rotating surfaces such as those in motors and cylinders, must not corrode components and must have a long life without chemical breakdown.

The earliest hydraulic systems used water for the fluid. While water is safe for humans and environment, cheap and readily available, it has significant disadvantages for hydraulic applications. Water provides almost no lubrication, has low viscosity and leaks by seals, easily cavitates when subjected to negative pressures, has a narrow temperature range between freezing and boiling ($0$ to $100\,^\circ$ C), is corrosive to the steels used extensively in hydraulic components and is a friendly environment for bacteria and algae growth, which is why swimming pools are chlorinated.

Modern hydraulic system use petroleum based oils, with additives to inhibit foaming and corrosion. Petroleum oils are inexpensive, provide good lubricity and, with additives, have long life. The brake and automatic transmission fluids in your car are examples.

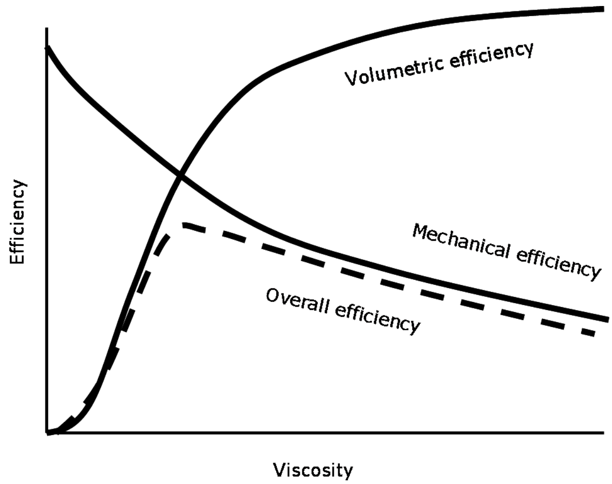



**Figure 2.4.:** Viscosity related hydraulic system losses.

## 2.4. Fluid Behavior

### 2.4.1. Viscosity

All fluids, including oil and air have fundamental properties and follow basic fluid mechanics laws. The *viscosity* of a fluid is its resistance to flow. Some fluids, like water, are thin and have low viscosity while others like honey are thick and have high viscosity. The fluids for hydraulic systems are a compromise. If the viscosity is too low, fluid will leak by internal seals causing a volumetric loss of efficiency. If the viscosity is too high, the fluid is difficult to push through hoses, fittings and valves causing a loss of mechanical efficiency. Figure 2.4 shows this tradeoff and indicates that a medium viscosity fluid is best for hydraulic applications.

The *dynamic viscosity* (also known as the absolute viscosity) is the shearing resistance of the fluid and is measured by placing the fluid between two plates and shearing one plate with respect to the other. The symbol for dynamic viscosity is the Greek letter mu ($\mu$). The SI unit for dynamic viscosity is the pascal-second (Pa-s), but the more common unit is the centipoise (cP), with 1 cP $=$ 0.001 Pa-s. The dynamic viscosity of water at 20 $^\circ$ C is 1.00 cP

It is easier to measure and more common to report the *kinematic viscosity* of a fluid, the ratio of the viscous forces to inertial forces. The symbol for kinematic viscosity is the Greek letter nu ($\nu$). Kinematic viscosity can be measured by the time it takes a volume of oil to flow through a capillary. The SI unit for kinematic viscosity is $m^2/s$ but the more com-

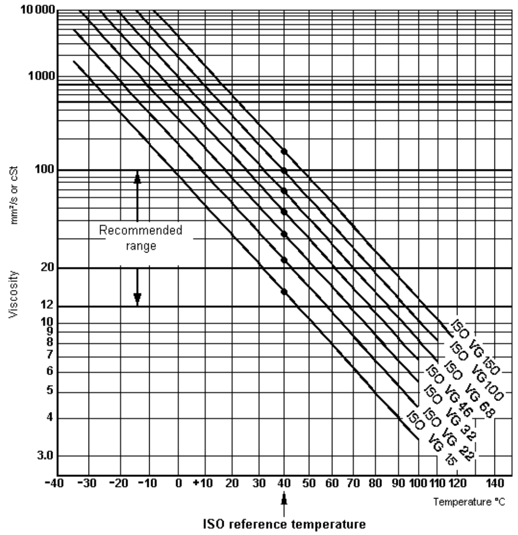



**Figure 2.5.:** The viscosity of hydraulic oils varies with temperature.

mon unit is the centistoke (cSt) which is 1 mm$^2$/s. The conversion is $1\text{m}^2/\text{s} = 10^6\text{cSt} = 10^4$stokes

If $\rho$ is the fluid density, the kinematic and dynamic viscosity are related by

$$\nu = \frac{\mu}{\rho} \tag{2.2}$$

The kinematic viscosity of water over a wide range of temperature is 1 cSt while common hydraulic oils at $40\,^\circ$ C are in the range of 20-70 cSt. Sometimes hydraulic oil kinematic viscosity is expressed in Saybolt Universal Seconds (SUS), which comes from the oil properties being measured on a Saybolt viscometer.

Viscosity changes with temperature; as fluid warms up it flows more easily. One reason your car is hard to start on a very cold morning is that the engine oil thickened overnight in the cold. The viscosity index VI expresses how much viscosity changes with temperature. Fluids with a high VI are desirable because they experience less change in viscosity with temperature. Figure 2.5 shows how viscostiy changes for typical hydraulic fluids.

## 2.4.2. Bulk Modulus

In many engineering applications, liquids are assumed to be completely incompressible even though all materials can be compressed to some

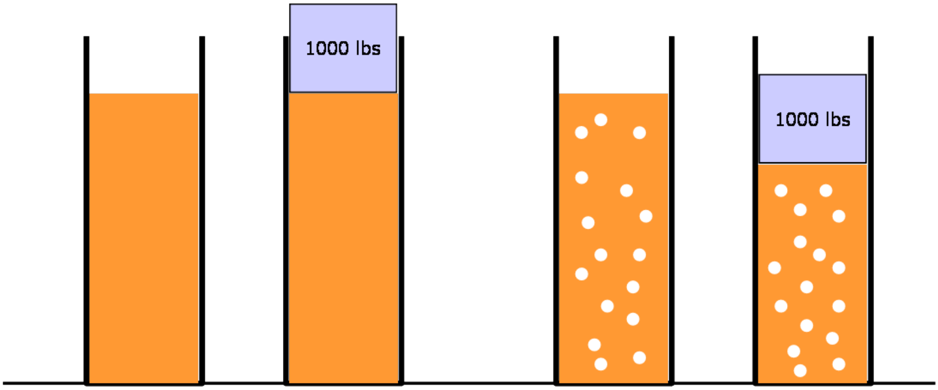



**Figure 2.6.:** Fluid bulk modulous. Liquids are nearly incompressible (left), except when there is trapped air, as shown on the right.

degree. In some hydraulic applications, the tiny compressibility of oil turns out to be important because the pressures are high, up to 5,000 psi. The *bulk modulus* of the fluid is the property that indicates the springiness of the fluid and is defined as the pressure needed to cause a given decrease in volume. A typical oil will decrease about 0.5% in volume for every 1000 psi increase in pressure. When the compressibility is significant, it is modeled as a fluid capacitor (spring) and often is lumped in with the fluid capacitance of the accumulator (see Section 3.4).

When air bubbles are entrained in the hydraulic oil, the bulk modulus drops and the fluid becomes springy (Fig. 2.6). You may have experienced this when the brake pedal in your car felt spongy. The solution was to bleed the brake system, which releases the trapped air so that the brake fluid becomes stiff again.

Another way that the fluid can change properties is if the pressure fall below the vapor pressure of the liquid causing the formation of vapor bubbles. When the bubbles collapse, a shock wave is produced that can erode nearby surfaces. Cavitation damage can be a problem for propellers and for fluid power pumps with the erosion greatly shortening the lifetime of components.

The bulk modulus $\beta$ is defined as

$$\beta = \frac{\Delta P}{\Delta V / V} \tag{2.3}$$

where $V$ is the original volume of liquid and $\Delta V$ is the change in volume of the liquid when subjected to a pressure change of $\Delta P$. Because $\Delta V/V$ is dimensionless, the units of $\beta$ are pressure. Water has a bulk modulus of $3.12 \times 10^5$ psi (2.15 GPa) while hydraulic oils have a bulk modulus between $2 \times 10^5$ and $3 \times 10^5$ psi (SAE 30 oil is $2.2 \times 10^5$ psi = 1.5 GPa), but can drop way down if air is entrained.

---

**Example 2.4.1.** A piston pushes down on a volume of SAE 30 oil trapped in a cylinder like the one shown in Figure 2.6. The cylinder bore is 5 in. and the height of the unstressed column of oil is 4 in. The oil has a bulk modulus $\beta = 2.2 \times 10^5$ psi. When a force of 10,000 lbs is applied to the piston, how much does the piston displace?

*Solution*: Use Equation 2.3 to calculate the change in volume

$$\begin{aligned}\Delta V &= \frac{\Delta P V}{\beta} = \frac{(F/A)(Ah)}{\beta} = \frac{Fh}{\beta} \\ &= \frac{(1 \times 10^4) \times 4}{2.2 \times 10^5} \\ &= .18 \text{ cu. in.}\end{aligned}$$

The change in height is

$$\begin{aligned}\Delta h &= \frac{\Delta V}{A} \\ &= \frac{.18}{\pi(5/2)^2} \\ &= 0.0092 \text{ in.}\end{aligned}$$

---

### 2.4.3. Pascal's Law

Pascal's Law states that in a confined fluid at rest, pressure acts equally in all directions and acts perpendicular to the confining walls (Fig. 2.7). This means that all chambers, hoses and spaces in a fluid power system that have open passageways between them are at equal pressure so long as the fluid is not moving.

Pascal's Law makes it easy to understand the operation of a simple hydraulic amplifier. Figure 2.8 shows how it works. The piston on the right is 25 times the area of the piston on the left. Pushing down with 10 lbs on the left piston will lift a 250 lb load on the right because the pressure of 10 psi is the same everywhere.

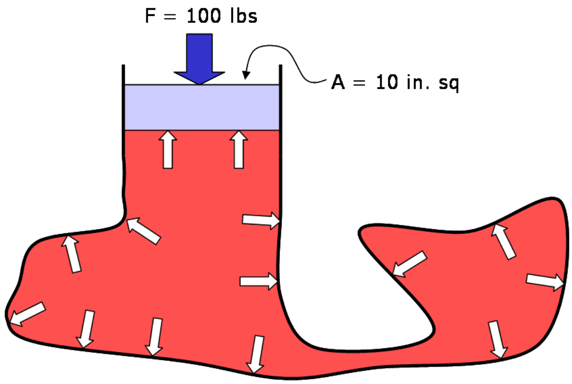



**Figure 2.7.:** Pascal' Law. Fluid at rest has same pressure everywhere and acts at right angles against the walls.

### 2.4.4. High Forces

The crowning glory of fluid power is the ability for small, light fluid power cylinders to produce extremely large forces. Consider the Caterpillar 345D hydraulic excavator shown in Figure 2.9. The bore (diameter) of the cylinder that drives the stick is 7.5 in. and the maximum working pressure is 5511 psi. The peak force in the cylinder is therefore

$$F = P \times A = 5511 \times \pi \times (7.5/2)^2 = 243,468 \text{ lbs}$$

or over 120 tons! It is impossible to produce this force using electric motors.

Force can be increased two ways, raising the pressure or increasing the bore of the cylinder. Force is proportional to pressure but goes as the square of the bore, which means small increases in cylinder size results

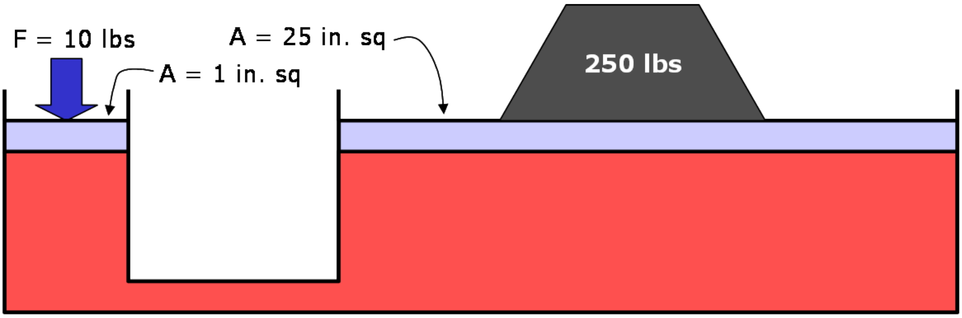



**Figure 2.8.:** Hydraulic amplifier. Pushing down with 10 lbs on the left piston raises the 250 lb. weight on the right piston.

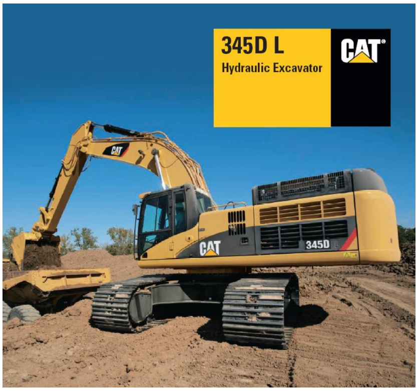



**Figure 2.9.:** A large excavator. Source: Caterpillar.

in large increases in force. For example, consider a small pneumatic cylinder with a 7/16 in. bore running at 60 psi. This cylinder can push with 9 lbs of force. But, as shown in Figure 2.10, force goes up as bore squared and a 4 in. bore cylinder at the same 60 psi can push with 754 lbs of force.

Of course the extra force does not come free. A larger cylinder requires a greater volume of fluid to move the load than a smaller cylinder, the practical result of the requirement that power be conserved.

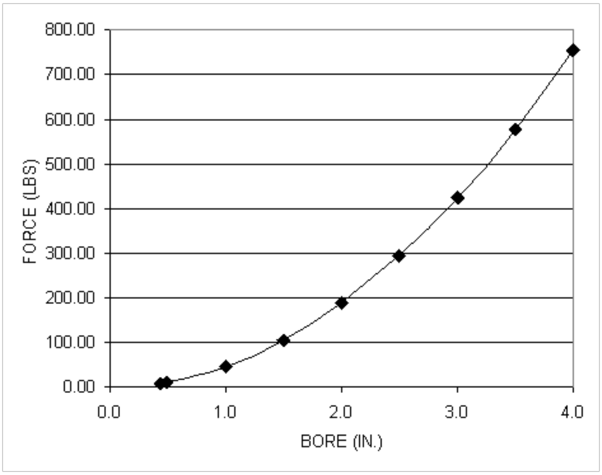



**Figure 2.10.:** Force in a cylinder acting at 60 psi. goes up with the square of the bore.

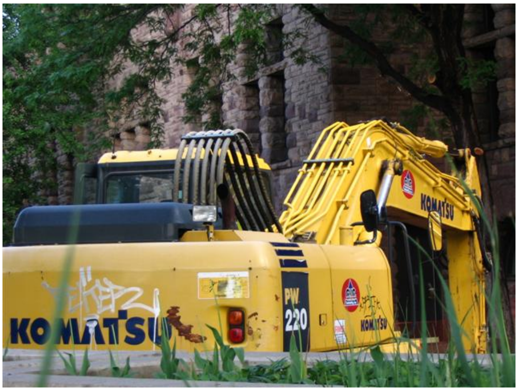



**Figure 2.11.:** Hoses on an excavator.

## 2.5. Conduit Flow

In electromechanical actuation systems, power is carried to motors through appropriately sized, low-resistance wires with negligible power loss. This is not the case for fluid power systems where the flow of oil through hydraulic hoses and pipes can result in energy losses due to internal fluid friction and the friction against the walls of the conduit (Fig. 2.11). Designers size the diameter and length of hydraulic hoses to minimize these losses. The other cause of major losses in fluid power systems is the orifice drag of valves and fittings. These will be discussed in Section 3 These loses are modeled as nonlinear resistances in the hydraulic circuit.

Conduit flow properties can be analyzed using basic principles of fluid mechanics. Typical flow patterns through pipes are shown in Figure 2.12. At low velocities, the flow is smooth and uniform while at higher velocities the flow turns turbulent. Turbulent flow can also be caused by sudden changes in direction or when the area suddenly changes, conditions that are common in hydraulic systems. Turbulent flow has higher friction, which results in greater heat losses and lower operating efficiencies. This is a practical concern for designers of hydraulic systems because every right angle fitting designed into the system lowers the system efficiency.

The Reynolds number, the non-dimensional ratio of inertial to viscous forces, is commonly used to characterize the flow in pipes

$$\mathrm{Re} = \frac{\rho V D_h}{\mu} = \frac{V D_h}{\nu} \tag{2.4}$$

where $\rho$ is the density (kg/m$^2$), $V$ is the mean fluid velocity (m/s), $D_h$ is the hydraulic diameter (m), $\mu$ is the dynamic viscosity (Pa-s) and $\nu$ is the

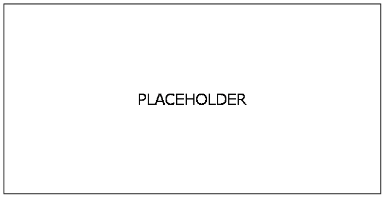



**Figure 2.12.:** Laminar and turbulent flow through a pipe.

kinematic viscosity. For circular conduits, the hydraulic diameter is the same as the pipe diameter. The more general formula, which accounts for non-circular pipes and hoses is

$$D_h = \frac{4A}{S} \tag{2.5}$$

where $A$ is the cross-sectional area of the conduit and $S$ is the perimeter. For a circle this formula reduces to diameter of the circle.

For fully developed pipe flow, if $\mathrm{Re} < 2100$ the flow is laminar, if $2100 < \mathrm{Re} < 4000$ the flow is in transition, neither laminar or turbulent, and if $\mathrm{Re} > 4000$ the flow is turbulent. These are approximations as the actual flow type will depend on local conditions and local geometry, which is why you may find other sources using different Reynolds number break points to define the flow type.

---

**Example 2.5.1.** A hydraulic hose with internal diameter of 1.0 in. is carrying oil with kinematic viscosity 50 cSt at a flow rate of 20 gpm. Calculate the Reynolds number and determine if the flow is laminar or turbulent.

*Solution*: Convert to SI units.

$$\begin{aligned} Q &= 20 \text{ gpm} = .00126 \text{ cu-m/s} \\ D &= 1.0 \text{ in.} = .0254 \text{ m} \\ \nu &= 50 \text{ cSt} = 5.0 \times 10^{-5} \text{ m}^2/\text{s} \end{aligned}$$

Find the average velocity

$$V = \frac{4Q}{\pi D^2} = \frac{(4)(.00126)}{(3.142)(.0254)^2} = 2.49 \text{ m/s}$$

Calculate the Reynolds number

$$\mathrm{Re} = \frac{\mathrm{VD}}{\nu} = \frac{(2.49)(.0254)}{5.0 \times 10^{-5}} = 1265$$

Because Re is below 2000 the flow is laminar.

---

### 2.5.1. Pressure Losses in Conduits

The pressure losses in straight pipe and hoses contribute to the overall efficiency of a fluid power system. The losses can be estimated using the Darcy-Weisbach equation

$$\Delta P = f\frac{\rho L}{2D}V^2 \tag{2.6}$$

where $\Delta P$ is the pressure drop, $f$ is the friction factor, $L$ is the pipe length, $D$ is the pipe inside diameter and $V^2$ is the average flow velocity. There are different formulas for the friction factor depending on the Reynolds number and the surface roughness of the pipe or hose.

The experimental determination of friction factors are shown on the classic Moody Diagram [1] (Fig. 2.13) that plots the friction factor as a function of Reynolds number for a range of surface roughness of round pipes. Surface roughness is defined as $\epsilon/D$ where $\epsilon$ is the mean height of the roughness of the pipe. The diagram shows that for laminar flow, friction factor is independent of roughness. For turbulent flow, the friction factor, and therefore pressure losses, depend both on the Reynolds number and the roughness of the pipe, except at high Reynolds numbers where losses depend only on the roughness. Once the Reynolds number is determined, simple formulas can be used to calculate pressure drop that avoid having to estimate numbers off the Moody Diagram.

**Laminar Flow** For laminar flow, the friction factor depends only on the Reynolds number

$$f = \frac{64}{Re} \tag{2.7}$$

Using this equation, Equation 2.6 and Equation 2.4 the pressure loss in a pipe with laminar flow can be calculated as

$$\Delta P = \frac{32\mu L}{D^2}V \tag{2.8}$$

Converting average velocity to the more convenient fluid flow rate ($V = 4Q/\pi D^2$) yields

$$\Delta P = \frac{128\mu L}{\pi D^4}Q \tag{2.9}$$

[1] Moody, L. F. (1944), "Friction factors for pipe flow", *Transactions of the ASME* 66 (8): 671–684

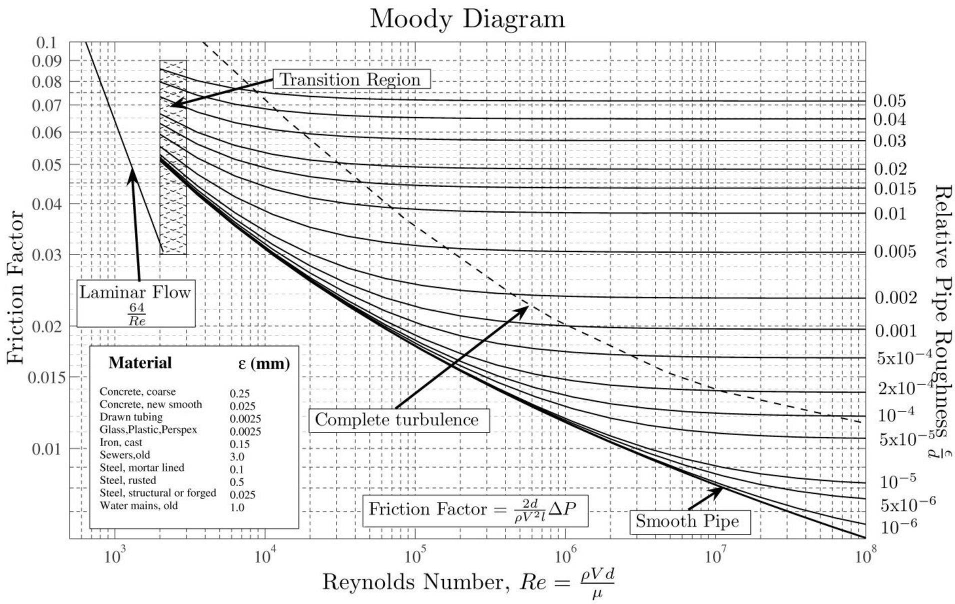



**Figure 2.13.:** Moody Diagram showing the experimental friction factor for pipes.

This equation shows that for smooth flow, the pressure drop is proportional to the flow rate $Q$. This is analogous to a linear resistor in an electrical system and to a linear damper in a mechanical system.

**Turbulent Flow, Smooth Pipes** For turbulent flow, the friction factor depends both on the Reynolds number and the surface roughness of the pipe, however, in most fluid power systems the pipes and hoses have smooth interiors and the friction factor for smooth pipes can be used. Under these conditions, an empirical approximation, derived by Blasius and based on experimental data, can be used to calculate the friction factor

$$f = \frac{0.316}{Re^{0.25}} \tag{2.10}$$

Combining with Equation 2.4 and converting from velocity to flow, the pressure loss in a smooth pipe with turbulent flow is

$$\Delta P = 0.214 \frac{\mu^{.25} \rho^{.75} L}{D^{4.75}} Q^{1.75} \tag{2.11}$$

The pressure loss is almost, but not quite, proportional to the square of the flow rate, thus a pipe with turbulent flow acts like a nonlinear resistor.

**Turbulent Flow, Rough Pipes** If needed, the turbulent friction factor data for rough piles in the Moody diagram can be modeled by the Colebrook Equation [2]

$$\frac{1}{\sqrt{f}} = -2\log_{10}\left(\frac{\epsilon/D_h}{3.7} + \frac{2.51}{\mathrm{Re}\sqrt{f}}\right)$$

This equation must be solved iteratively for $f$, but a close-approximation direct solution that can be used for hydraulic system modeling purposes comes from the Swamee-Jain Equation [3]

$$f = \frac{0.25}{\left[\log_{10}\left(\frac{\epsilon}{3.7D} + \frac{5.74}{\mathrm{Re}^{0.9}}\right)\right]^2}$$

---

**Example 2.5.2.** Hydraulic oil ISO 68 is flowing through a hydraulic line with inside diameter 2.0 in. at a rate of 200 gpm. Find the pressure drop in psi for a 10 ft length of hose.

*Solution*: Hydraulic oil ISO 68 has a density of 54.9 lb/cu-ft (880 kg/cu-m) and a kinematic viscosity $\nu$ of 68.0 cSt at $104^\circ$ F and 10.2 cSt at $212^\circ$ F. For this problem, assume the oil is at $104^\circ$ F. First, convert to SI units.

$$\begin{aligned}
Q &= \frac{6.31 \times 10^{-5}\ \text{cu-m/s}}{1\ \text{gpm}} \times 200\ \text{gpm} = .0126\ \text{cu-m/s} \\
D &= \frac{1\ \text{m}}{39.37\ \text{in.}} \times 2.0\ \text{in.} = .0504\ \text{m} \\
L &= \frac{1}{3.281} \times 10 = 3.048\ \text{m} \\
\nu &= \frac{1\ \text{m}^2/\text{s}}{10^6\ \text{cSt}} \times 68.0 = 6.8 \times 10^{-5}\ \text{m}^2/\text{s} \\
\rho &= 880\ \text{kg/cu-m} \\
\mu &= \rho\nu = (880)(6.8 \times 10^{-5}) = .0598\ \text{Pa-s}
\end{aligned}$$

Calculate the average velocity

$$V = \frac{4Q}{\pi D^2} = \frac{(4)(.0126)}{(3.142)(.0504)^2} = 6.316\ \text{m/s}$$

[2] Colebrook, C.F. (1938), "Turbulent Flow in Pipes", J Inst Civil Eng 11:133.
[3] Swamee, P.K.; Jain, A.K. (1976). "Explicit Equations for Pipe-Flow Problems". *Journal of the Hydraulics Division*, ASCE 102 (5): 657664

Calculate the Reynolds number

$$\text{Re} = \frac{\text{VD}}{\nu} = \frac{(6.316)(.0504)}{6.8 \times 10^{-5}} = 4681$$

This indicates turbulent flow. Using Equation 2.11 for turbulent flow and a smooth pipe gives the pressure drop

$$\begin{aligned}
\Delta P &= 0.214\frac{(.0598^{.25})(880^{.75})(3.048)}{(.0504^{4.75})}(.0126^{1.75}) \\
&= 35,980 \text{ Pa} \\
&= \frac{1 \text{ psi}}{6895 \text{ Pa}} \times 35,980 \\
&= 5.2 \text{ psi}
\end{aligned}$$

Because most hydraulic systems operate we above 500 psi, the 5.2 psi drop in the 10 ft. line is inconsequential. The pressure drop scales linearly with line length so longer hoses, and certainly smaller hoses, could impact system efficiency.

---

*Note*: Calculations such as the one in the last example can be tedious and prone to errors because it is easy to lose track of units. If you find yourself doing these calculations often, consider developing a spreadsheet macro that takes care of the details. Software for hydraulic line calculations is also available [4] and fluid power simulation software automatically calculate line losses, a big plus if you are designing a large system.

## 2.6. Bends and Fittings

Flow through long straight pipes is not common in practical fluid power system as typically flow goes through right-angle fittings and short sections of bent, flexible hose. For example, Figure 2.14 shows an SAE 90 deg elbow fitting made to SAE/JIC standards. The pressure drops in these pathways can be significant and must be estimated to analyze system efficiency. While fluid mechanics theory and computational fluid dynamics simulations can be used to generate precise values of pressure loss, the fluid power engineer generally uses tabulated values of

[4] For example, a free web-based calculator is at *www.efunda.com/formulae/fluids/calc_pipe_friction.cfm* and a comprehensive hydraulic sizing software tool that includes line loss calculations is available for a small cost from *www.CompuDraulic.com*.

**Figure 2.14.:** SAE hydraulic elbow fitting.

dimensionless *loss coefficients* $K$ for each component. The pressure drop across the component is

$$\Delta P = K\frac{\rho}{2}V^2 = K\frac{\rho}{2A^2}Q^2 \tag{2.12}$$

where $A$ is the area of the fitting. Note that the pressure drop is proportional to the flow squared. Inverting Equation 2.12 gives the flow as a function of pressure

$$Q = \frac{A}{\sqrt{K}}\sqrt{\frac{2}{\rho}\Delta P} \tag{2.13}$$

Loss coefficients can be found in vendor catalogs. Table 2.3 lists $K$ for some common fittings and flow path geometries. [5] For accurate modeling, loss coefficients must be found experimentally.

**Table 2.3.:** Loss coefficients $K$ for geometric elements

| Fitting | $K$ |
|---:|---:|
| $90\,^\circ$ deg elbow | 0.2 |
| $45\,^\circ$ deg elbow | 0.15 |
| Tee fitting | 0.9 |
| Sharp-edged entrance | 0.5 |
| Rounded entrance | 0.05 |
| Sharp-edged exit | 1.0 |
| Rounded exit | 1.0 |

[5] For a 45 and 90 deg elbows, the loss coefficients depend on the ratio of the radius of the bend to the diameter of the fitting. Texts such as [1] and [2] have reference information.

## 2.7. Orifice Flow

The third major pressure loss comes from the flow of fluid through the restricted orifices found in valves and some fittings. Here the classic orifice equation, valid for steady, incompressible flow with Re $>> 1$, is used to model the flow

$$Q = C_d A \sqrt{\frac{2\Delta P}{\rho}} \tag{2.14}$$

or rewriting for the pressure drop

$$\Delta P = \frac{\rho}{2A^2 C_d^2} Q^2 \tag{2.15}$$

where $A$ is is cross-section of the orifice and $C_d$ is the discharge (or valve) coefficient. The discharge coefficient can be a variable, changing with valve position, however, an average value for $C_d$ of 0.62 is often used to simplify calculations leaving area $A$ to change with valve position.

The fixed parameters can be lumped together to form a slightly different version of the orifice loss equation

$$Q = C_v \sqrt{\frac{\Delta P}{SG}} \tag{2.16}$$

or

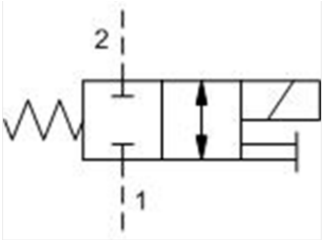
$$\Delta P = \frac{SG}{C_v^2} Q^2 \tag{2.17}$$

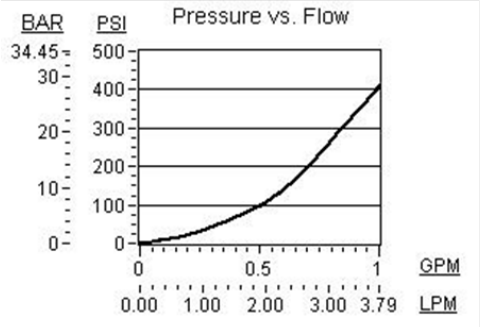
where $SG$ is the dimensionless specific gravity for the fluid (ratio of the density of the fluid to the density of water) and $C_v$ is the valve coefficient. When U.S. manufacturers state a value for $C_v$ in a valve data sheet, it assumes that $Q$ is in gpm and $\Delta P$ is in psi and that the temperature and viscosity are fixed. Typical hydrocarbon based oils have specific gravities between 0.85 and 0.95. In all cases, note that flow is proportional to the square root of the pressure drop across the valve, which makes it a nonlinear resistance.

---

**Example 2.7.1.** Find $C_v$ for a Sun Hydraulics DAAA solenoid-operated on-off valve when running hydraulic oil with a specific gravity of 0.864.

*Solution*: The Sun Hydraulics DAAA valve is a 2-position, 2-way cartridge valve. When the solenoid is energized the valve switches from blocking the flow to a fully open flow. The data sheet is at *www.sunhydraulics.com*. Some of the key valve specs are: Max flow = .25 gpm (1 L/m), Max pressure = 5000 psi (350 bar), Response time = 30 ms. The figure below shows a photo of the valve, the valve schematic and the experimental pressure vs. flow characteristics for the valve in its open position. These are cartridge valves, designed to screw into a manifold with the solenoid on top.




The following approximate (gpm,psi) data points can be read off the graph: (0,0), (0.3,50), (0.5,100), (1,410). With the help of a spreadsheet the points were fit to an approximate parabola, psi $= 400\text{gpm}^2$. Using Equation 2.17

$$
\begin{aligned}
\frac{SG}{C_v^2} &= 400 \\
C_v &= \sqrt{\frac{SG}{400}} \\
&= \sqrt{\frac{.864}{400}} \\
&= 0.046
\end{aligned}
$$

# 3. Fluid Power Components

This chapter describes the most common components found in typical fluid power systems. Basic mathematical models for the constitutive properties of components are presented, which are the building blocks for understanding how to select components and for simulating fluid power circuits. Standard symbols for the schematic representation of flud power components are also introduced. Fluid power symbols are referenced in ISO standard 1219-1:2006, which is explained in Appendix A. While this section will not make you an expert in practical hydraulic and pneumatic systems, it will build your understanding of the fundamental principles underlying such systems.

## 3.1. Cylinders

Cylinders convert fluid power pressure and flow to mechanical translational power force and velocity (Fig. 3.1). Cylinders are linear actuators that can push and pull, and when mounted around a joint, for example, as is done in an excavator, can actuate rotary motion. Cylinders come in single acting (push only), single acting with spring return and double-acting (push-pull). The rest of the section will focus on double acting cylinders, which are most common in hydraulic applications.

A cutaway illustration of a typical double-acting cylinder used for industrial applications is shown in Figure 3.2 and the ISO symbol is shown in Figure 3.3

The end of the cylinder where the rod emerges is called the *rod end*

**Figure 3.1.:** Hydraulic cylinder.

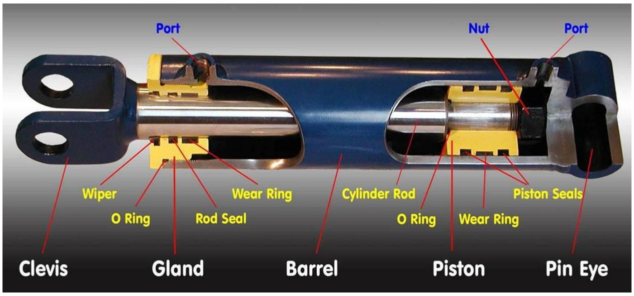



**Figure 3.2.:** Cut-away view of a welded body, industrial double-acting hydraulic cylinder. Source: Hyco International

and the other end is called the *cap end*. This distinction is important for modeling because the rod side of the piston within the cylinder has less surface area than the cap side of the piston. For the same pressure a double-acting cylinder can push with much greater force than it can pull. Examination of the cylinder configuration in an excavator (e.g. Fig. 1.4 shows how designers can take advantage of the larger pushing force. Cylinders have considerable friction, particularly around the piston because of its large circumference with wrap-around seals. The rod seal tends to be even tighter than the piston seals to prevent leaking of hydraulic oil, but because of the smaller circumference, rod seals play less of a role in cylinder friction.

While the design of a cylinder is complex, the dynamic model used for most simulations simply captures the pressure-force transformation and sometimes includes the cylinder friction and leakage around the piston seal. The defining equations for an ideal, friction-free, leakless cylinder are

$$F = PA \tag{3.1}$$

$$V = Q/A \tag{3.2}$$

The piston force depends on the difference in pressure across the piston, taking into account the area on each side. Referring to Figure 3.4 the piston force is

$$F_P = P_1 A_{cap} - P_2 A_{rod} \tag{3.3}$$

where all pressures are gauge pressures with respect to atmospheric

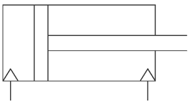


**Figure 3.3.:** ISO schematic symbol for a double-acting cylinder.

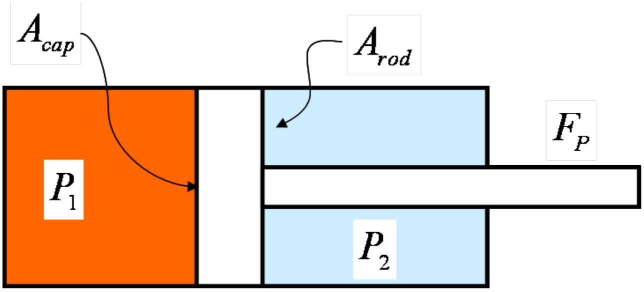



**Figure 3.4.:** Forces on a cylinder.

pressure. [1] and the piston velocity is

$$V = \frac{Q_1}{A_{cap}} = \frac{Q_2}{A_{rod}} \tag{3.4}$$

The rod area is the piston annulus around the rod and is

$$A_{rod} = \frac{\pi}{4}\left(\text{bore}^2 - \text{roddia}^2\right)$$

Another consequence of the different areas on either side of the piston is that the oil flow in one port will not be equal to the flow out the other port. If the return line to the reservoir is long or the return line valve has small orifices then the pressure build up on the rod side of the cylinder when pushing can be significant and must be modeled.

The overall efficiency of a cylinder is given by the ratio of the output mechanical power to the input fluid power

$$\eta = \frac{FV}{P_i Q_i} \tag{3.5}$$

where $P_i$ and $Q_i$ refer to either the cap or rod side depending on whether the piston is pushing or pulling. Cylinder efficiency can be split into two parts, the force efficiency

$$\eta_f = \frac{F}{P_i A_i} \tag{3.6}$$

and the volumetric efficiency

$$\eta_v = \frac{A_i V}{Q_i} \tag{3.7}$$

[1] For pneumatic systems where compressible gas laws define cylinder behavior, pressures are absolute, which means that the defining force equation is

$$F_P = P_1 A_{cap} - (P_2 A_{annulus} + P_{atm} A_r)$$

where $A_{annulus}$ is the area of the piston annulus around the rod and $A_r$ is the area of the piston rod.

with the overall efficiency being the product $\eta = \eta_f \eta_v$. Using these relations, Equations 3.3 and 3.4 can be modified for a non-ideal cylinder with friction and leakage

$$F_P = (P_1 A_{cap} - P_2 A_{rod})\, \eta_f \tag{3.8}$$

$$V = \frac{Q_1}{A_{cap}} \eta_v = \frac{Q_2}{A_{rod}} \eta_v \tag{3.9}$$

## 3.2. Pumps and Motors

Hydraulic pumps supply energy to the system, converting the torque and velocity of an input shaft to pressure and flow of the output fluid. Hydraulic motors are the rotary equivalent of cylinders. Pressurized oil flows through the motor and produces rotation of the output shaft. One way of looking at a motor is simply as a pump driven in reverse (fluid flow in, shaft rotation out versus shaft rotation in and fluid flow out) and indeed many devices can act both as a pump and a motor, much the same way that a DC permanent magnet motor can act both as a motor and a generator. The common types of pumps and motors are gear and vane pumps and motors and axial and radial piston pumps and motors motors. Fixed displacement devices rotate a fixed amount for a fixed volume of fluid. Variable displacement devices have a mechanical or fluid power control port that can be used to vary the relation between fluid volume and shaft rotation, and are essentially a variable transmission between fluid and mechanical rotary power. Pumps are generally driven by electric motors in industrial applications and diesel engines in mobile applications. [2] The symbols for fixed displacement, uni-directional pumps and motors are shown in Figure 3.5.

Ideal pumps and motors are defined by the relations between fluid pressure $P$ and flow $Q$ and shaft torque $T$ and velocity $\omega$. For an ideal motor, input and output power is conserved. If $\Delta P$ is the pressure difference across the pump or motor, then the power balance for a pump or motor is

$$Power = \Delta P Q = T\omega \tag{3.10}$$

If the volumetric displacement of the motor or pump is $D_v$, typically expressed in cubic in./rad, then the relations relating fluid to mechanical

[2] For descriptions of how various pumps and motors are work and their applications, see these web resources:
http://en.wikipedia.org/wiki/Hydraulic_pump
http://www.hydraulicspneumatics.com/200/TechZone/HydraulicPumpsM/74

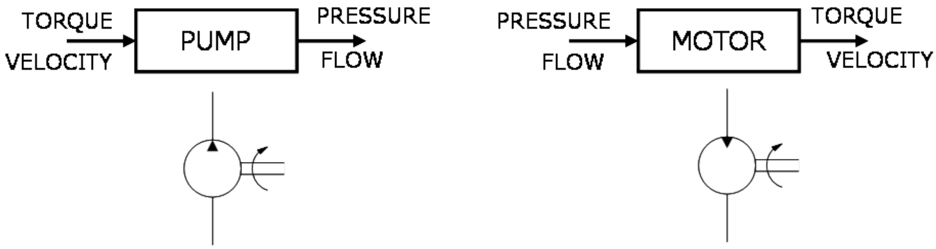



**Figure 3.5.:** Pumps, motors and their schematic symbols. The difference between the two symbols is the direction that the triangle is pointing; out for pumps, in for motors

are

$$T = D_v \Delta P \tag{3.11}$$

$$Q = D_v \omega \tag{3.12}$$

The parameter $D_v$ is the sole parameter that defines the operating characteristics of an ideal pump or motor and in that respect is similar to the torque constant of a DC motor. From the above equations, it can be seen that a pump can be a constant flow source if its shaft is rotated at constant velocity and a constant pressure source if the shaft torque is fixed. In practice, neither is the case as the shaft is rotated by an electric motor and the motor-pump combination results in pressure-flow performance curves. An example is shown in Figure 3.6.

Real pumps and motors are not 100% efficient and, like cylinders, have an overall efficiency $\eta$, which is made up of volumetric $\eta_v$ and mechanical $\eta_m$ efficiencies with

$$\eta = \eta_v \eta_m \tag{3.13}$$

Table 3.1 shows the equations for the various efficiencies for pumps and for motors.

**Table 3.1.:** Efficiency equations for pumps and motors

| Pump | Motor |
|---|---|
| $\eta = \Delta PQ/T\omega$ | $\eta = T\omega/\Delta PQ$ |
| $\eta_v = Q/D_v\omega$ | $\eta_v = D_v\omega/Q$ |
| $\eta_m = D_v\Delta P/T$ | $\eta_m = T/D_m\Delta P$ |

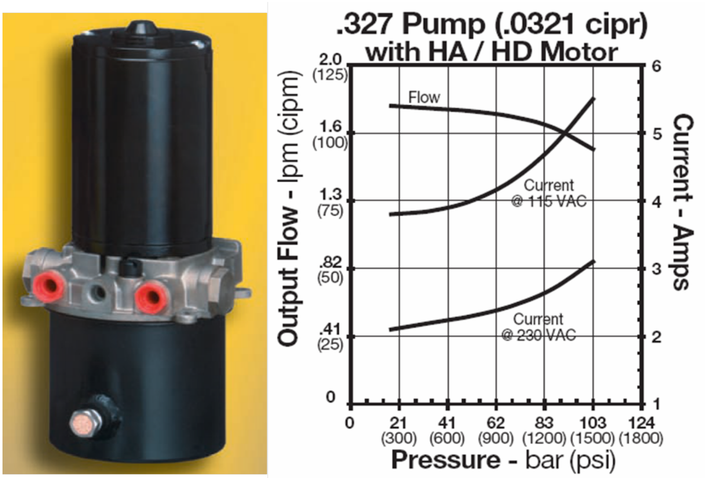



**Figure 3.6.:** A small, packaged hydraulic power supply containing a 115V AC electric motor, hydraulic pump, relief valve, reservoir tank and filter. The combined electric motor plus pump results in the output pressure-flow characteristics shown in the data sheet to the right. Source: Oildyne.

## 3.3. Control Valves

Control valves are essential and appear in all fluid power systems. Valves are sometimes categorized by function, which includes directional control valves for directing fluid flow to one or the other side of a cylinder or motor, pressure control valves for controlling the fluid pressure at a point and flow control valves for limiting the fluid flow rate in a line, which in turn limits the extension or retraction velocities of a piston.

Valves are also characterized by the number of ports on the valve for connecting input and output lines and by the number of operating positions that the valve can assume. For example, a 3-way, 2-position valve commonly found in pneumatic systems has three ports for connecting supply line, exhaust or reservoir line and output line to the cylinder and two positions. In one position the supply line connects to the cylinder line extending the piston. In the other position the exhaust line connects to the cylinder retracting the piston, assuming the piston has a spring return. On/off valves can only be in the states defined by their positions while proportional valves are continuously variable and can take on any position in their working range. A servo valve is a proportional valve with an internal closed-loop feedback mechanism to maintain precise control over the valve behavior. Example valves are shown in Figure 3.7

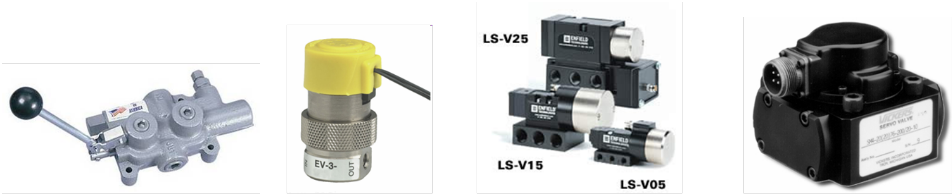



**Figure 3.7.:** Types of control valves. Left to right: hand-operated directional valve for a log splitter. On-off miniature, solenoid actuated pneumatic valve. Precision proportional pneumatic valves. High precision, flapper-nozzle hydraulic servo valve.

### 3.3.1. Dynamic Models for Valves

For dynamic modeling purposes, valves are fundamentally variable orifices where the area of the orifice depends on the valve position. For example, the core dynamic model of a solenoid proportional valve has the area of an orifice as a nonlinear function of the command signal to the solenoid.

The basic equation for a valve is the orifice equation introduced in Section 2.7 and repeated here

$$Q = C_d A \sqrt{\frac{2\Delta P}{\rho}} \tag{3.14}$$

where $C_d$ is valve coefficient, $A$ is the area of the valve opening and $P$ is the pressure drop across the valve. For valves with internal spools and rectangular orifice slots, the orifice opening area is proportional to the valve position, $A = wx$.

To simplify analysis, the orifice equation can be linearized about a nominal operating point at the $x = 0$ valve position with leakage flow $Q_0$

$$Q = Q_0 + K_q x + K_c P \tag{3.15}$$

The linearized valve is characterized by two parameters, the *flow gain coefficient*

$$K_q = \frac{\partial Q}{\partial x} = C_d \sqrt{\frac{2P}{\rho}} \frac{\partial A}{\partial x} = C_d w \sqrt{\frac{2P}{\rho}} \tag{3.16}$$

and the *flow pressure coefficient*

$$K_c = \frac{\partial Q}{\partial P} = \frac{A C_d}{\sqrt{2P\rho}} = \frac{w x C_d}{\sqrt{2P\rho}} \tag{3.17}$$

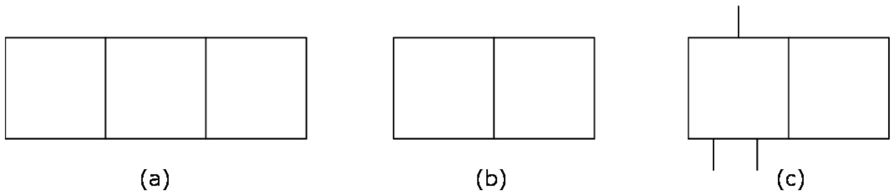



**Figure 3.8.:** Basic symbology for fluid power valves. The number of squares is the number of valve positions. Image (a) represents a three-position valve and (b) represents a two-position valve. The number of connection points on the symbol is the number of ports ("'ways"') for the valve. Image (c) represents a 3-way, 2-position valve or "'3/2 valve"' for short.

### 3.3.2. Valve Symbols

Because there are so many types of valves, their symbols can be complex. Some of the more basic symbols are covered in this section.

A valve has one square for each working position. The nominal or initial valve position has connection points, or ways, to the valve ports. Thus a three-way two-position (3/2) valve would have three connection ports and two boxes as shown in Fig. 3.8. The ports are sometimes labeled with letters with A,B,C,... indicating working lines, P indicating the pressurized supply line and T or R indicating the return (tank) line connected to the reservoir.

Lines with arrows inside the boxes indicate the path and direction of flow. Pneumatic systems are indicated with unfilled arrow heads. Examples are shown in Figure 3.9.

The icons on the side of the symbol indicate how the valve is actuated. Common methods include push button, lever, spring-return, solenoid (for computer-control) and pilot-pressure line. Examples are shown in Figures 3.10 and 3.11. Also shown in Figure 3.11 is an example of a

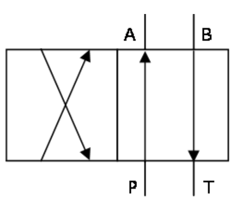



**Figure 3.9.:** A 4/2 valve with four connection points and two positions. In the nominal, unactuated state, supply line P connects to working line A and working line B connects to return (tank) line T. In the actuated state, the valve slides to the right and supply line P connects to working line B and working line A connects to return line T.

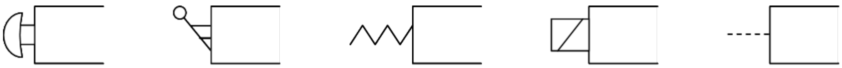


**Figure 3.10.:** Valve actuation symbols. Left to right: push-button, lever, spring-return, solenoid, pilot-line.

proportional valve that can take on any position. An infinite position, proportional valve is indicated by parallel lines on top and bottom of the symbol.

## 3.4. Accumulators

Hydraulic accumulators are used for temporarily storing pressurized oil. The oil enters a chamber and acts against a piston or bladder to raise a weight, compress a spring or compress a gas. Accumulators are used to supply transient peak power, which reduces the flow rate requirement for the power supply and to act as shock absorbers for smoothing out pressure wave spikes. Accumulators are the equivalent to a capacitor in an electrical system and to a spring in a mechanical system. Bladder type accumulators, precharged with nitrogen gas are the most common type for hydraulic systems (Fig. 3.12).

The capacity of a fluid capacitor is defined by its change in volume divided by its change in pressure

$$C_f = \frac{\Delta V}{\Delta P} \tag{3.18}$$

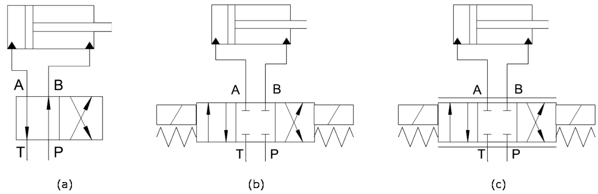



**Figure 3.11.:** (a) Pushbutton hydraulic 4/2 valve. In the nominal position, the cylinder is held in the retracted position because supply line P is connected to the rod side. When the button is pushed, the rod extends because supply line P is now connected to the cap side. When the button is released, the valve spring returns the valve to the nominal position, retracting the rod. (b) Solenoid 4/3 valve with center closed position and spring return to center. A computer can control extension and retraction of the cylinder by actuating the valve solenoids. (c) Same as b, but with a continuously variable proportional valve.

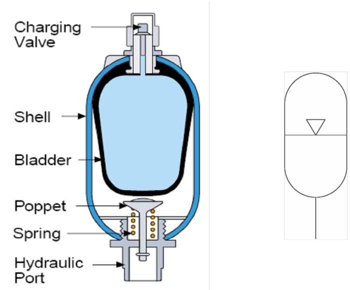



**Figure 3.12.:** Accumulator and ISO symbol.

Change in volume per time is flow rate and change in pressure per time is the derivative of pressure. This leads to the constituative law for a linear fluid capacitor

$$Q = C_f \dot{P} \tag{3.19}$$

For a gas-filled accumulator, the capacitance $C_f$ will depend on the accumulator pre-charge. The capacitance is the slope of the accumulator volume-pressure curve, which is sometimes given in the manufacturer's data sheet. If the curve is nonlinear, the slope at the operating point should be taken for $C_f$.

Another type of accumulator is a cylinder with the fluid pushing on one side of the piston against a stiff spring on the other side of piston. For these spring-loaded piston accumulators the capacitance is

$$C_f = \frac{A^2}{K} \tag{3.20}$$

where $A$ is the area of the piston and $K$ is the spring constant.

## 3.5. Filters

During use, hydraulic oil picks up contaminating particles from wear of sliding metallic surface that add to residual contaminants from the oil manufacturing process, rust from metal and polymer particles from seal wear. These dirt particles are tiny grit that cause additional abrasive wear. Clumps of particles can clog tiny clearances in precision valves and cylinders and can lead to corrosion.

All practical hydraulic systems require a filter in the circuit (Fig. 3.13). In-line filters have a fine mesh media formed from wire, paper or glass fiber, formed to create a large surface area for the fluid to pass through. The oil filter in your car is an example of a hydraulic filter. Sometimes the filter is included inside the reservoir or is part of an integrated power supply unit along with the motor, pump and reservoir. Selecting a filter

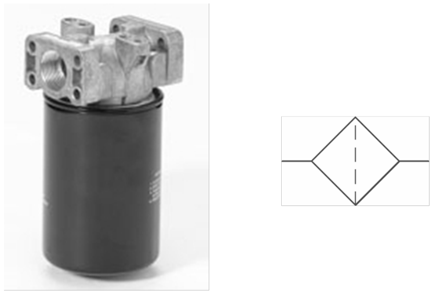


**Figure 3.13.:** Hydraulic filter and ISO symbol.

is a tradeoff between a media that traps fine contaminants and one that passes fluid with minimal resistance.

The dynamic model for a filter is a nonlinear resistance $P = f(Q)$ that can be linearized about the nominal flow. If the pressure drop across the filter is small compared to other pressure drops in the system, the effects of the filter on the dynamic model can be ignored. Resistance values for simulation models can be estimated from the manufacturer's data sheet or from a filter characterization experiment.

## 3.6. Reservoirs

The main function of the reservoir is to provide a source of room temperature oil at atmospheric pressure (Fig. 3.14). The reservoir is equivalent to the ground in an electrical system. Conceptually, a reservoir is nothing more than an oil storage tank connected to atmosphere through a breather and having pump and return lines to deliver and accept oil. In practice, a reservoir has additional functions including de-aerating and acting as a heat exchanger. The dynamic model of a reservoir is to treat it as a ground, a source of zero pressure.

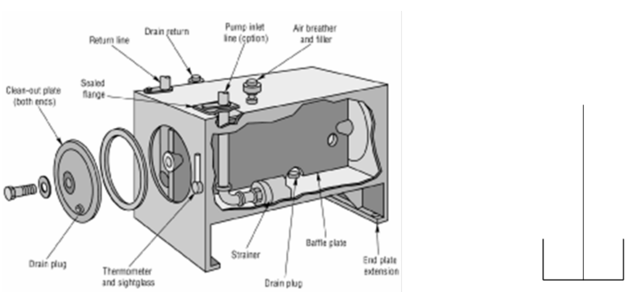



**Figure 3.14.:** Hydraulic reservoir and ISO symbol.

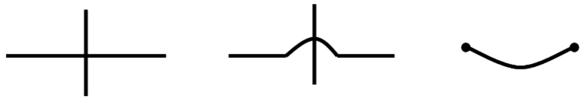


**Figure 3.15.:** Symbols for fluid power lines. Left to right: lines joined, lines crossing, flexible line

## 3.7. Hoses and Fittings

The glue that connects the various components together are the hydraulic hoses and fittings. As described in Sections 2.5, 2.6 and 2.7, they are modeled as fluid power resistors with with linear or non-linear pressure-flow characteristics. Symbols for pipes and hoses are shown in Figure 3.15

# 4. Hydraulic Circuit Analysis

A basic hydraulic circuit is shown in Figure 4.1. It contains a motor-driven, fixed-displacement hydraulic pump, a 4-way, 3-position, center off valve and a double-acting hydraulic cylinder. A pressure relief valve is stationed between the pump output and the return line. This valve is required because otherwise, with the valve shut and no place for the supply fluid to move, the output pressure of the fixed displacement pump would quickly build up to dangerous levels. A typical setting for the relief valve for a small system is 600 psi. The combination of the fixed displacement pump and the relief valve effectively turns the combination into a constant pressure supply, assuming the pump flow rate can keep up with the load demands. While the pump plus relief valve combination is common, it is not efficient because when the system is idling, the pump is wasting energy pushing flow through the pressure valve drop back to the tank.

Hydraulic circuit static and dynamic analysis involves first developing the appropriate mathematical models for each component in the circuit and then using Pascal's Law (pressure same at all points for fluid at rest) and conservation of flow to connect the components into a set of equations that describes the complete system. For a static analysis, the set of equations will be algebraic while a dynamic analysis a set of differential equations.

For example, the circuit shown in Figure 4.1 could be modeled as a

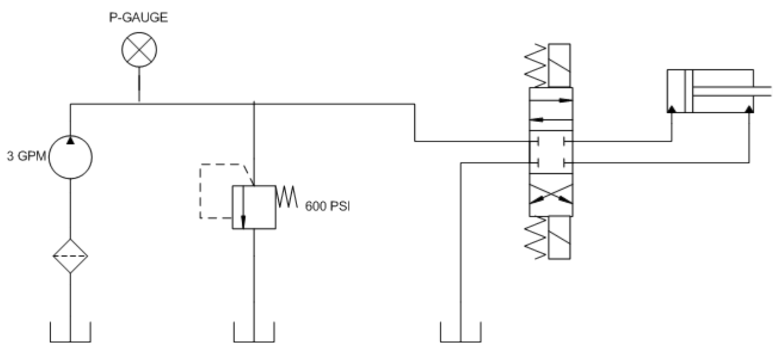



**Figure 4.1.:** A basic hydraulic circuit with fixed-displacement pump, pressure relief valve, 4/3 solenoid valve and cylinder.

constant source of pressure feeding into the valve because of the combination of the positive displacement pump coupled to the relief valve. The pressure would be the setpoint of the relief valve, for example 600 psi. The valve could be modeled as a nonlinear resistance whose value depends on the position of the valve spool as shown in Section 3.3.1. The pressure drop in the lines would also be modeled as a nonlinear resistor as described in Section 2.5.1. The cylinder is modeled as a transformer that converts pressure and flow to force and velocity (Section 3.1). Not shown is the mechanical load that the cylinder acts against, which would be some combination of inertia, friction, damping and spring. The return lines from the cylinder through the valve and back to the reservoir would be modeled as a resistors in series. If the filter introduced significant pressure losses, it would also be modeled as a resistor. Because the pressures in typical hydraulic systems are so high (1000 to 3000 psi), small pressure drops in hoses and filters are often neglected or treated in rule-of-thumb tables when sizing power supplies. Accurate models are required, however, to understand efficiencies and detailed system behavior.

A steady-state static analysis of the circuit would entail writing equations for a network of nonlinear resistors and the force balance across the piston. These equations can be solved to determine the pressures and flows at various points in the circuit. A dynamic analysis with differential equations is needed if the cylinder pushed against a load with springs or inertias, if there were an accumulator in the circuit or if fluid capacitance were significant.

For dynamic analysis, particularly for the purpose of designing a high performance hydraulic control system, the system is generally linearized so that linear control design methods can be used. The process for linearizing a control valve was described in Section 3.3.1.

The rest of this section covers the building blocks needed to develop dynamic models, resistance, capacitance, inertance and source elements, and presents several modeling examples.

## 4.1. Fluid Resistance

Fluid resistors are any component that resists flow. Another way of looking at fluid resistors are any component that causes a pressure drop when fluid flows through the component. Fluid resistors include valves, filters, hoses, pipes and fittings. A generalized linear fluid resistor relates flow and pressure

$$P = R_f Q \tag{4.1}$$

or

$$Q = \frac{1}{R_f} P \tag{4.2}$$

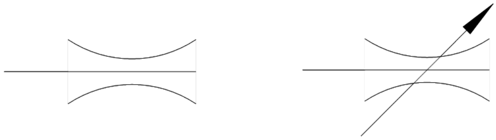


**Figure 4.2.:** Symbol for a fixed (left) or variable (right) restrictor valve. This symbol is sometimes used to indicate a parasitic resistance, for example the flow resistance in a pipe.

As we saw in the sections on conduit, orifice and valve flow, real fluid resistors are nonlinear and typically relate flow to the square root of pressure. The general form of resistance can be written as

$$P = f_R(Q) \tag{4.3}$$

or

$$Q = f_r^{-1}(P) \tag{4.4}$$

In fluid power schematics, sometimes a general resistance, for example to represent the flow resistance in a pipe, is indicated with the restrictor valve symbol (Figure 4.2).

## 4.2. Fluid Capacitance

Fluid capacitors are one of two types of energy storing elements in fluid power systems. Capacitance in a fluid power circuit comes from discrete accumulators (Section 3.4 but also from the fluid itself if it is compliant. Fluid compliance is essential to consider in pneumatic systems, but generally does not play a significant role in dynamic models of hydraulic systems unless there is significant trapped air causing spongy behavior.

The capacitance of the fluid is captured by its bulk modulus property (Section 2.4.2. The capacitance of a trapped section of compliant fluid can be determined as shown in the following example.

---

**Example 4.2.1.** Find the equivalent capacitance of the hydraulic ram for the system described in Example 2.4.1

*Solution*: From the solution to Example 2.4.1 the fluid volume changed 1.8 cu. in. for a pressure change of $10,000/19.63 = 509$ psi. The capacitance of the fluid trapped in the cylinder is

$$C_f = \frac{1.8}{509} = 3.54 \times 10^{-3}$$

The idea of capacitance of fluid trapped in a cylinder can be expanded to estimate the capacitance of a plug of fluid in a hose or pipe, which in turn can be used in a dynamic model. One application of a dynamic model involving fluid compression is to understand water hammer, which is impact loading cause by sudden changes in flow, such as when a valve is switched from on to off.

Equation 3.18 states that capacitance is the change in volume divided by change in pressure while Equation 2.3 defines the bulk modulus $\beta$ as the change in pressure divided by the normalized change in volume. These can be combined into an expression for the capacitance of a known volume of fluid, for example the fluid in a length of pipe.

$$C_f = \frac{\Delta V}{\Delta P} \tag{4.5}$$

$$= \frac{V}{\beta} \tag{4.6}$$

---

**Example 4.2.2.** Find the fluid capacitance of SAE 30 oil with bulk modulus $\beta = 2.2 \times 10^5$ psi. flowing through in a 20 inch length of 2 inch diameter hose.

*Solution*: The volume of the fluid is

$$V = \frac{L\pi D^2}{4}$$

Using Equation 4.6 the capacitance is

$$\begin{aligned} C_f &= \frac{V}{\beta} = \frac{L\pi D^2}{4\beta} \\ &= \frac{(20)(3.14)(4)}{(4)(2.2 \times 10^5)} \\ &= 2.85 \times 10^{-4} \end{aligned}$$

---

In a fluid power network, fluid capacitance that comes from an accumulator is referenced to ground (zero gauge pressure) while capacitance from compressibility of the fluid is referenced to the pressures at the two ends of the fluid plug.

## 4.3. Fluid Inertance

The second type of energy storing element is fluid inertance. In mechanical systems, mass and rotary inertia often dominate system behavior and must be modeled. In fluid power systems, the inertia of the fluid is generally insignificant and usually ignored in dynamic system models. The reason is that in hydraulic systems, pressures are so high that inertial forces can be neglected and in pneumatic systems the mass of air is so low that inertial forces can also be neglected.

When analyzing high frequency behavior of a system, for example with sudden on off switching of valves that causes transients in fluid flow, fluid inertance should be included in the model. The inertance of a plug of fluid in a hose is simply the mass of the fluid

$$I_f = \frac{\rho L}{A} \tag{4.7}$$

where $\rho$ is the fluid density and $L$ and $A$ are the length and area of the hose.

In a network, the inertance of fluid in a pipe is always modeled as being in series with the flow resistance of the pipe.

## 4.4. Connection Laws and States

### 4.4.1. Connections

When hydraulic components are connected, conservation of flow and pressure loop principles are used to write the equations that join components. Figure 4.3 shows these simple rules.

### 4.4.2. State Variables

For generating system equations, the state variables are the pressures $P$ at the nodes and the flows $Q$ through the elements. These are completely analogous to voltage $V$ and current $I$, the states for electrical systems.

For dynamic system models, the order and the number of first order differential equations that describe the system is equal to the number of independent energy storage elements. If there are no fluid capacitances or fluid inertances in the system then the order of the system is zero and the equations will be purely algebraic with no derivatives.

## 4.5. Example Systems

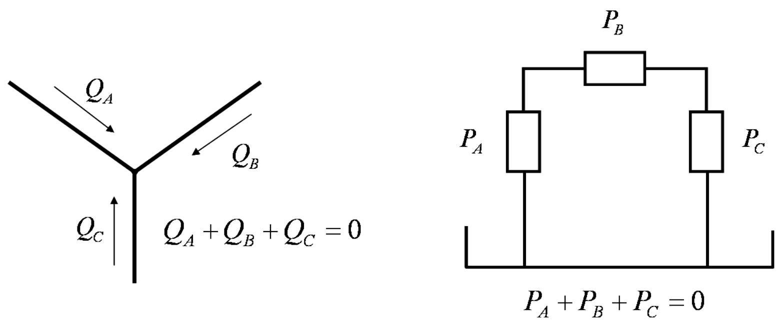
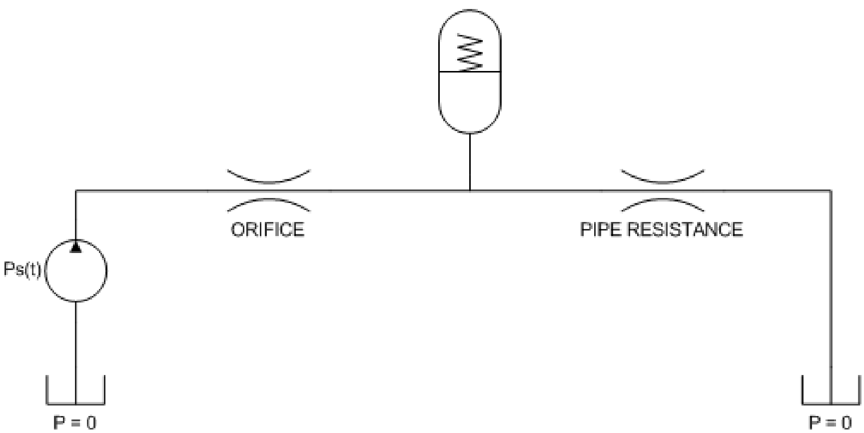



**Figure 4.3.:** Flow and pressure connection laws. The flows into a node must sum to zero. The pressure drops around a loop must sum to zero. In the figure on the right, $P_i$ indicates the pressure drop across component $i$.

**Example 4.5.1.** Write the state equations for the system shown in the figure. There are two resistances, one is an orifice the other is the pipe resistance. The output of interest is the pressure at the accumulator.




*Solution*: The pressure at the reservoir is 0 and the pump is modeled as a pressure source with output pressure $P_S$. There is only one other pressure in the system, $P_A$, the pressure in the accumulator. Write the element and connection equations.

**Orifice** The behavior of the orifice is described by Equation 2.14 or Equation 2.16. For this analysis, lump all constants into one paramter $K_o = C_v\sqrt{1/SG}$ so that $Q = K_o\sqrt{P_{\text{orifice}}}$. From continuity, the flow through the orifice is $Q_S$

$$P_{\text{orifice}} = P_S - P_A$$

$$Q_S = K_o\sqrt{P_S - P_A} \tag{4.8}$$

**Pipe Resistance** Assume turbulent flow. The full expression for turbulent flow in a smooth pipe is given by Equation 2.11. For this analysis, make the approximation that the pressure loss is proportional to flow squared rather than to flow to the 1.75 power. Use $K_P$ for the proportional constant. The flow through the pipe is the output flow $Q_o$ and the pressure across the pipe is $P = P_A - 0 = P_A$. Thus, the pipe resistance is described by

$$Q_o = K_P\sqrt{P_A} \tag{4.9}$$

**Accumulator** The accumulator is the only energy storage element in the system and is described by Equation 3.19

$$Q_A = C_f\dot{P}_A \tag{4.10}$$

where $Q_A$ is the flow into the accumulator.

**Continuity** Conservation of flow dictates that

$$Q_S = Q_A + Q_o \tag{4.11}$$

**State Equation** The goal is to find a set of state equations (for this first-order example with one energy storage element there will be one equation) with $P_A$ as the output and $P_S$ as the input. Using (4.10), (4.9) and (4.11) yields

$$C_f\dot{P}_A = Q_S - Q_o = Q_S - K_P\sqrt{P_A} \tag{4.12}$$

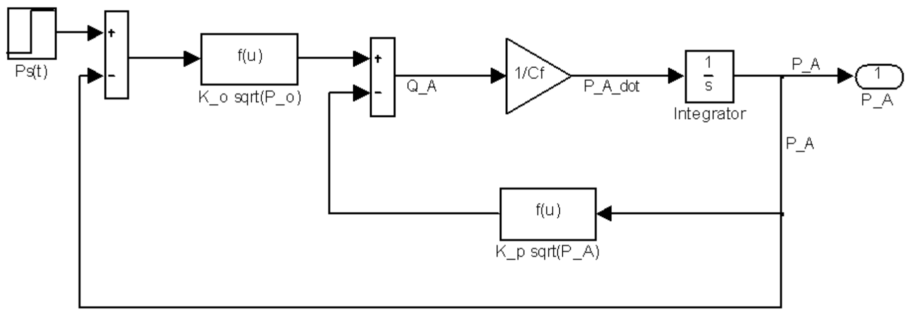
Using (4.8) and (4.12)

$$C_f\dot{P}_A = K_o\sqrt{P_S - P_A} - K_P\sqrt{P_A} \tag{4.13}$$

Dividing by $C_f$ yields the state equation

$$\dot{P}_A = \frac{K_o}{C_f}\sqrt{P_S - P_A} - \frac{K_P}{C_f}\sqrt{P_A} \tag{4.14}$$

Because the state equation is nonlinear, it cannot be solved directly but can be easily simulated numerically in a package such as Simulink. A Simulink block diagram for this example is shown below.


Ps(t)
+
-
f(u)
K_o sqrt(P_o)
+
-
Q_A
1/Cf
P_A_dot
1
s
Integrator
P_A
1
P_A
P_A
f(u)
K_p sqrt(P_A)

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# A. Fluid Power Symbols

Fluid power symbols are set by International Organization for Standardization (ISO) standards, ISO 1219-1:2006 for fluid power system and component graphic symbols and ISO 1219-2:1995 for fluid power circuits.

The following tables show the basic ISO/ANSI symbols for fluid power components and systems. [1]

*Tables will appear in the next version. For now, lists of symbols can be found at these web locations:*

http://www.hydraulicsupermarket.com/upload/db_documents_doc_19.pdf
http://www.patchn.com/index.php?option=com_content&task=view&id=31&Itemid=31
http://www.hydraulic-gear-pumps.com/pdf/Hydraulic%20Symbols.pdf
http://www.hydrastore.co.uk/products/Atos/P001.pdf
http://www.scribd.com/doc/2881790/Fluid-Power-Graphic-Symbols

[1] Microsoft VISIO has a library of symbols for generating fluid power schematics, although may not be in the latest ISO format.